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"A Fuzzy Queuing Model with Unreliable Server, Fuzzy Arrival and Service Rates, and Fuzzy Service Time."

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Abstract

This paper presents a fuzzy queuing model that incorporates uncertainty in arrival and service rates, service time, and service reliability. The model is formulated as a fuzzy M/M/1 queuing system with an unreliable server, where the arrival rate, service rate, and service time are represented by fuzzy numbers. The server is assumed to be unreliable, with a fuzzy failure rate and fuzzy repair rate. Using fuzzy arithmetic and fuzzy calculus, We derive the fuzzy steady - state probabilities and performance metrices, such as the fuzzy expected number of customers in the system, fuzzy expected waiting time, and server utilization.

Numerical examples are provided to illustrate the application of the model and the effects at fuzzy parameters on the performance metrices. The results show that the fuzzy queuing model can effectively capture the uncertainty and imprecision in the real world queuing system, providing a more realistic and robust analysis.

Keywords: *- Fuzzy queuing model, unreliable server, fuzzy arrival and service rates, fuzzy service time, fuzzy arithmetic, fuzzy calculus.*

INTRODUCTION

Fuzzy logic was initiated by Zadeh. Fuzzy queuing model was first introduced by R.J.Lie and E.S.Lee in 1989, further developed this model by many authors. Later, Deveraj.J, and Jayalakshmi.D, Developed a fuzzy approach to priority queues in 2012. Mueen.Z, Ramli.R, and Zabidi.N, investigate analysis of performance measures with single channel fuzzy queues under two class by ranking method in 2017. Also, Kumar.R, and Som .B.K. developed a finite

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single server queuing system with reverse reneging in 2015. Later, Rao.S.H, Vemuri.V.K, Rao.T.S, and Kumar.B.S, investigate the Analysis of both arrival two-ohase MX/M/1 queuing system with impatient customers and unreliable server in 2018. Aria.M, investigate a new fuzzy logic system for controlling multiple traffic intersections with dynamic phase selection and pedestrian crossing signal in 2019. Rajalakshmi.R and Julia Rose Mary.K investigated the fuzzy analysis for M/M/C finite capacity queuing model in 2017. Usha Prameela.K and Pavan Kumar developed the single transmit fuzzy queuing model with two-classes by the execution proportins by ranking technique in 2020. Hepshiba.B and Julia Rose Mary.K investigate the application of queuing model for Healthcare centre in 2020. A fuzzy queuing model with an unreliable server incorporates fuzzy set theory to handle uncertainties in real-world scenarios where parameters like arrival rates and service times are imprecise or vary based on linguistic descriptions

PRELIMINARIES

Definition 2.1: Let Y be a universal set. Then the fuzzy subset B of Y is defined by its membership function Y, where the value of ∊B(y) in the interval to each element xμ which assign a real number →B: Y μ B(y) at y shows the grade of membership of y in B. The membership function of a fuzzy set is known as μa possibility distribution.

Definition 2.2: Given a fuzzy set B in Y and any real number β then the β-cut or β-level or cut worthyset of B, denoted by βB is the crisp set. The strong denoted by β+B is the crispset. For example, let B be a fuzzy set whose membership function is given as,

$$\mu_B(y) = \begin{cases} \frac{y-b}{c-b}, & if\, b \le y \le c \\ \frac{d-y}{d-c}, & if\, c \le y \le d \end{cases}$$

= (y – b) / β[0,1] to both left and right reference functions of B. That is, ∊ To find the β-cut of B, we first set β, asβ = (c –y) / (d – b). That is, y can be expressed in terms of β(c – b) and ∊]. yβ +b, d- (d-c)βB = [(c – b)β-cut of B as β which gives β+b and y = d- (d-c)βy = (c – b)[b,c] is called the left reference function, which is right continuous, monotone and non-decreasing,

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while the y [c, d] is called right reference function, which is left continuous, monotone and non- increasing. The above∈ definition of a fuzzy number is known as an L-R fuzzy number.

TECHNIQUES FOR SOLVING PROBLEMS OF QUEUEING MODELS

Queueing models are classified as Markovian queueing models and non-Markovian queueing models. The techniques generally adopted to solve these types of queueing models are explained below.

Markovian Queueing Models

Queueing models with exponential interarrival time and exponential service time are called Markovian queueing models. Some of the techniques used to solve Markovian queueing models are:

1. Difference – differential equation method
2. Neuts matrix-geometric algorithm
3. Continued fraction method

Some queueing systems are studied analytically by deriving the corresponding difference - differential equations and solving them by applying Rouche's theorem through suitable generating functions. The first method is discussed elaborately by Gross and Harris (1998) and Saaty (1961). Neuts (1981) developed the matrix-geometric algorithmic approach to study the steady state queueing models. This method involves real arithmetic and avoids the calculation of complex roots based on Rouche's theorem.

Unreliable Server

An unreliable server is subject to random breakdowns and repairs.

- **Breakdown Rate :** The fuzzy rate at which the server fails. Breakdown can occur while the server is idle or actively serving a customer.
- **Repair Rate :** The fuzzy rate at which the server is restored to operation.
- **Delayed Repair :** In some models, the repair does not start immediately; the server may wait for a "setup time" or for a specific number of customers to accumulate before restoration begins.

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Conclusion

In short, the paper concludes that integrating fuzzy arithmetic and fuzzy calculus into queuing theory provides a more sophisticated tool for decision-makers. It bridge the gap between theoretical mathematics and the "fuzzy" or "imprecise" nature of actual service environments

If you'd like, I can help you with more specific details, such as:

Explaining the fuzzy M/M/1 system in simpler terms.

Defining the specific fuzzy arithmetic used (like triangular or trapezoidal fuzzy numbers).

Comparing this to a standard queuing model without fuzzy logic.

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